

WPI



Graduate School of Information Sciences
Tohoku University

VISLab@WPI

Loss Distillation via Gradient Matching for Point Cloud Completion with Weighted Chamfer Distance

Fangzhou Lin, Haotian Liu*, Haoying Zhou*, Songlin Hou*, Kazunori Yamada, Gregory Fischer, Yanhua Li, Haichong Zhang, and Ziming Zhang*



- **Problems Statements:**
 - Chamfer Distance (CD) and variants, are sensitivity to outliers and need time-consuming parameter tuning.
 - Can we define a weighted CD to boost the performance of the vanilla CD?
 - If we can reproduce the exact gradients in training, we can then reproduce the performance of a certain loss (HyperCD in this case).
- **Motivation:**
 - We aim to explore functional spaces to search for good functions.
 - The distances in the metric as a training loss should be weighted in some form rather than uniform.
 - We borrow the idea from network distillation, rather than networks, we aim to learn losses functions instead.

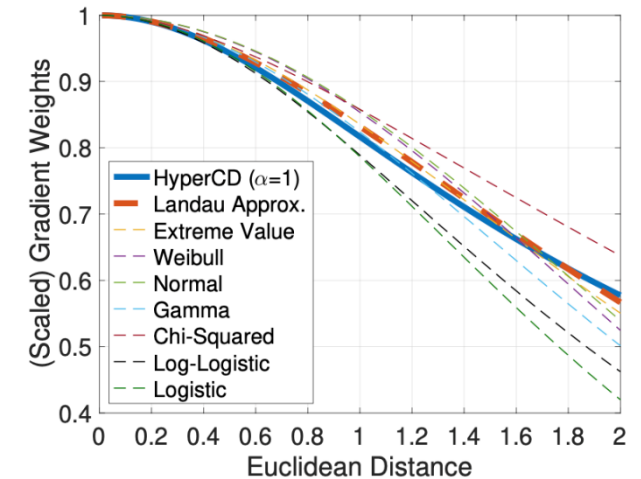


Fig 1. Illustration of distributions that are similar to the gradient weighting distribution from HyperCD and can be used as weighting functions in weighted CD.

- **Contribution:**
 - We propose a gradient-matching method for loss distillation.
 - We show strong performance for point cloud completion based on weighted CD
 - Our method can be simply done by simulated data with mathematical derivations.

- **Chamfer Distance:** CD serves as a popular training loss in point cloud completion for training neural networks.

— In our work:

$$D_{CD}(x_i, y_i) = \frac{1}{|x_i|} \sum_{j=1}^{|x_i|} \min_k d(x_{ij}, y_{ik}) + \frac{1}{|y_i|} \sum_{k=1}^{|y_i|} \min_j d(x_{ij}, y_{ik})$$

$$d(x_{ij}, y_{ik}) = \begin{cases} \|x_{ij} - y_{ik}\| & \text{as } L1\text{-distance} \\ \|x_{ij} - y_{ik}\|^2 & \text{as } L2\text{-distance} \end{cases}$$

- **Hyper CD:** Defines as follow equation

$$d(x_{ij}, y_{ik}) = \operatorname{arccosh} (1 + \alpha \|x_{ij} - y_{ik}\|^2), \alpha > 0.$$

- **Weighting Functions:** Distributions as weighting functions in WeightedCD

Distribution	Params	Mode m	PDF
Chi-Squared	k	$\max(k - 2, 0)$	$\frac{1}{2^{k/2} \Gamma(k/2)} x^{k/2-1} e^{-x/2}$
Extreme Value	β	0	$\frac{1}{\beta} e^{-(z+e^{-z})}, z = \frac{x}{\beta}$
Weibull	k, λ	$\begin{cases} \lambda \left(\frac{k-1}{k}\right)^{1/k}, & k > 1, \\ 0, & k \leq 1. \end{cases}$	$\frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$
Log-Logistic	α, β	$\begin{cases} \alpha \left(\frac{\beta-1}{\beta+1}\right)^{1/\beta}, & \text{if } \beta > 1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{\beta}{x} \left(1 + \left(\frac{x}{\alpha}\right)^{-\beta}\right)^{-1-\beta}$
Gamma	α, β	$\begin{cases} \frac{\alpha-1}{\beta}, & \text{for } \alpha \geq 1, \\ 0, & \text{for } \alpha < 1 \end{cases}$	$\frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$
Logistic	σ	0	$\frac{e^{-x/\sigma}}{\sigma(1 + e^{-x/\sigma})^2}$
Normal	σ	0	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$
Landau Approx.	-	0	$\frac{1}{\sqrt{2\pi}} \exp\left\{\left(-\frac{x + e^{-x}}{2}\right)\right\}$

Weighted CD

$$D_W(x_i, y_i) = \frac{1}{|x_i|} \sum_{j=1}^{|x_i|} f(\tilde{d}_{ijk}) \tilde{d}_{ijk} + \frac{1}{|y_i|} \sum_{k=1}^{|y_i|} f(\tilde{d}_{ikj}) \tilde{d}_{ikj} \quad (1)$$

$$\text{s.t. } \tilde{d}_{ijk} = \min_k d(x_{ij}, y_{ik}), \tilde{d}_{ikj} = \min_j d(x_{ij}, y_{ik}). \quad (2)$$

Loss Distillation via Gradient Matching:
Mimic the learning behavior of the target teacher CD (HyperCD in this case).

$$\min_{f \in \mathcal{F}} \mathbb{E}_{\tilde{d} \sim \tilde{\mathcal{D}}} \|z^{(H)}(\tilde{d}) - z^{(W)}(\tilde{d})\| \approx \min_{f \in \mathcal{F}} \sum_{\tilde{d}} p(\tilde{d}) \|z^{(H)}(\tilde{d}) - z^{(W)}(\tilde{d})\| \quad (3)$$

$$z^{(H)}(\tilde{d}) = \frac{2\alpha\tilde{d}}{\sqrt{(1+\alpha\tilde{d}^2)^2 - 1}}, z^{(W)}(\tilde{d}) = f'(\tilde{d})\tilde{d} + f(\tilde{d}) \quad (4)$$

Algorithm 1 Loss Distillation via Gradient Matching

Input : a PDF f with parameters \mathcal{A} , $z^{(H)}$, $\{(\tilde{d}, p(\tilde{d}))\}$

Output: \mathcal{A}

Discretize the parameter space into $\{\mathcal{A}_i\}$ for grid search;

Compute the mode m_i for each \mathcal{A}_i used in $z^{(W)}$;

$\mathcal{A}^* = \operatorname{argmin}_{\{\mathcal{A}_i\}} \sum_{\tilde{d}} p(\tilde{d}) \|z^{(H)}(\tilde{d}) - z^{(W)}(\tilde{d})\|$;

return $\mathcal{A} \leftarrow \mathcal{A}^*$

Searching Scheme

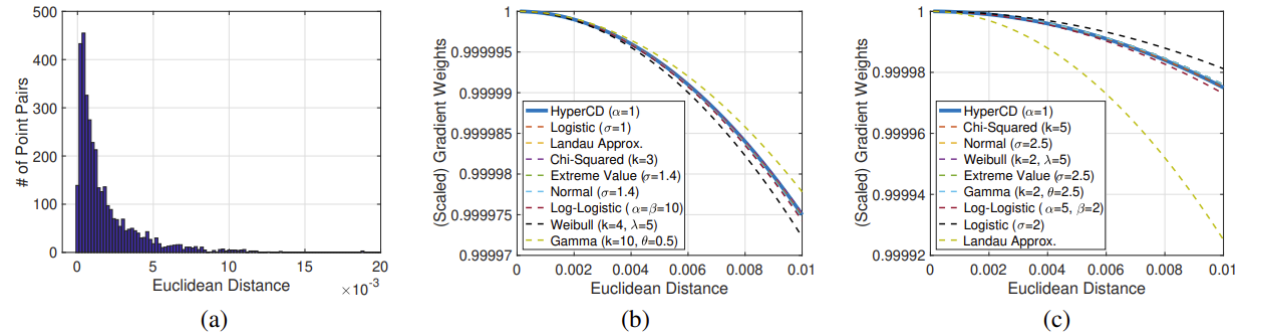


Fig 2. **(a)** Illustrate the distance distribution from HyperCD. In **(b-c)** respectively, where all the curves are rescaled by the maximum values and ordered by the minimum of Loss Distillation equation 3.

Point Cloud Completion with Weighted CD

Algorithm 2 Point Cloud Completion with Weighted CD

Input : a weighting function f , a network h with learnable parameters ω , training data $\{(\tilde{x}_i, y_i)\}$

Output: ω

repeat

 Pick a sample (\tilde{x}_i, y_i) uniformly at random;

 Compute $\tilde{d}_{ijk}, \forall j$ in \tilde{x}_i and $\tilde{d}_{ikj}, \forall k$ in y_i ;

 Compute the weighted CD loss based on Eq. 4;

 Update the parameters ω using the gradient of the loss;

until converges;

return ω

$$\min_{\omega} \sum_i \left[\frac{1}{|x_i|} \sum_{j=1}^{|x_i|} f(\tilde{d}_{ijk}) \tilde{d}_{ijk} + \frac{1}{|y_i|} \sum_{k=1}^{|y_i|} f(\tilde{d}_{ikj}) \tilde{d}_{ikj} \right]$$

$$\text{s.t. } x_i = h(\tilde{x}_i; \omega) = \{x_{ij}\}, \forall i,$$

$$\tilde{d}_{ijk} = \min_k d(x_{ij}, y_{ik}), \tilde{d}_{ikj} = \min_j d(x_{ij}, y_{ik}).$$

Results

- Completion Results:

- Evaluated on both synthesis: PCN, ShapeNet-55, and real-world datasets: KITTI. One of the proposed weightedCD, namely LandauCD, consistently outperformed HyperCD and other variants in multiple benchmarks.

Methods	Plane	Cabinet	Car	Chair	Lamp	Couch	Table	Boat	Avg.
FoldingNet [41]	14.31	9.49	15.80	12.61	15.55	16.41	15.97	13.65	14.99
TopNet [45]	7.61	13.31	10.90	13.82	14.44	14.78	11.22	11.12	12.15
AtlasNet [46]	6.37	11.94	10.10	12.06	12.37	12.99	10.33	10.61	10.85
GRNet [47]	6.45	10.37	9.45	9.41	7.96	10.51	8.44	8.04	8.83
CRN [48]	4.79	9.97	8.31	9.49	8.94	10.69	7.81	8.05	8.51
NSFA [49]	4.76	10.18	8.63	8.53	7.03	10.53	7.35	7.48	8.06
FBNet [50]	3.99	9.05	7.90	7.38	5.82	8.85	6.35	6.18	6.94
PCN [36]	5.50	22.70	10.63	8.70	11.00	11.34	11.68	8.59	11.27
FoldingNet [41]	9.49	15.80	12.61	15.55	16.41	15.97	13.65	14.99	14.31
HyperCD + FoldingNet	7.89	12.90	10.67	14.55	13.87	14.09	11.86	10.89	12.09
InfoCD + FoldingNet	7.90	12.68	10.83	14.04	14.05	14.56	11.61	11.45	12.14
Landau CD + FoldingNet	7.30	12.69	10.46	13.00	11.92	13.39	10.86	10.59	11.27
PMP-Net [42]	5.65	11.24	9.64	9.51	6.95	10.83	8.72	7.25	8.73
HyperCD + PMP-Net	5.06	10.67	9.30	9.11	6.83	11.01	8.18	7.03	8.40
InfoCD + PMP-Net	4.67	10.09	8.87	8.59	6.38	10.48	7.51	6.75	7.92
Landau CD + PMP-Net	4.59	10.10	8.90	8.57	6.38	10.47	7.49	6.75	7.92
PoinTr [11]	4.75	10.47	8.68	9.39	7.75	10.93	7.78	7.29	8.38
HyperCD + PoinTr	4.42	9.77	8.22	8.22	6.62	9.62	6.97	6.67	7.56
InfoCD + PoinTr	4.06	9.42	8.11	7.81	6.21	9.38	6.57	6.40	7.24
Landau CD + PoinTr	4.12	9.49	8.07	7.82	6.30	9.28	6.76	6.41	7.28
SnowflakeNet [12]	4.29	9.16	8.08	7.89	6.07	9.23	6.55	6.40	7.21
HyperCD + SnowflakeNet	3.95	9.01	7.88	7.37	5.75	8.94	6.19	6.17	6.91
InfoCD + SnowflakeNet	4.01	8.81	7.62	7.51	5.80	8.91	6.21	6.05	6.86
Landau CD + SnowflakeNet	3.98	8.97	7.78	7.40	5.76	8.86	6.16	6.14	6.88
PointAttN [14]	3.87	9.00	7.63	7.43	5.90	8.68	6.32	6.09	6.86
DCD + PointAttN	4.47	9.65	8.14	8.12	6.75	9.60	6.92	6.67	7.54
HyperCD + PointAttN	3.76	8.93	7.49	7.06	5.61	8.48	6.25	5.92	6.68
InfoCD + PointAttN	3.72	8.87	7.46	7.02	5.60	8.45	6.23	5.92	6.65
Gamma CD + PointAttN	3.83	8.96	7.58	7.15	5.69	8.56	6.34	6.01	6.76
Chi-Squared CD + PointAttN	3.77	8.93	7.49	7.08	5.64	8.50	6.28	5.95	6.70
Log-Logistic CD + PointAttN	3.78	8.92	7.47	7.10	5.63	8.51	6.29	5.94	6.70
Extreme-Value CD + PointAttN	3.73	8.88	7.46	7.03	5.61	8.46	6.25	5.92	6.66
Landau CD + PointAttN	3.72	8.88	7.46	7.04	5.60	8.47	6.24	5.93	6.66
SeedFormer [13]	3.85	9.05	8.06	7.06	5.21	8.85	6.05	5.85	6.74
DCD + SeedFormer	16.42	26.23	21.08	20.06	18.30	26.51	18.23	18.22	24.52
HyperCD + SeedFormer	3.72	8.71	7.79	6.83	5.11	8.61	5.82	5.76	6.54
InfoCD + SeedFormer	3.69	8.72	7.68	6.84	5.08	8.61	5.83	5.75	6.52
Log-Logistic CD + SeedFormer	3.86	9.07	7.79	6.89	5.15	8.64	5.87	5.78	6.63
Gamma CD + SeedFormer	3.84	9.01	7.82	6.89	5.13	8.63	5.88	5.75	6.61
Chi-Squared CD + SeedFormer	3.75	8.90	7.71	6.80	5.11	8.48	5.77	5.68	6.53
Extreme-Value CD + SeedFormer	3.73	8.88	7.70	6.80	5.08	8.48	5.75	5.65	6.51
Landau CD + SeedFormer	3.65	8.68	7.64	6.80	5.04	8.57	5.79	5.71	6.49

Table. 1 Comparison on PCN dataset in terms of per-point L1-CD x 1000 (Lower the better).

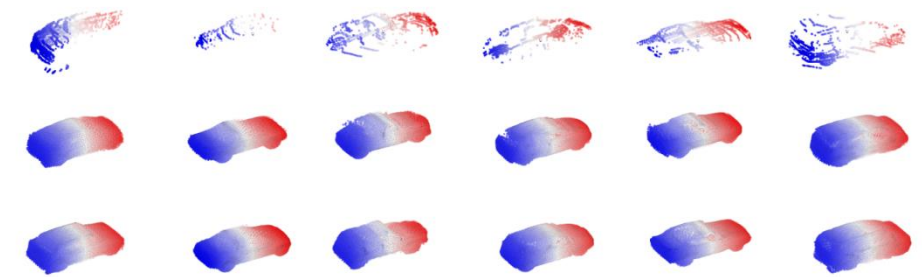


Fig. 3 Visualization of the KITTI benchmark, Row 1: input, Row 2: HyperCD, and Row 3: LandauCD.

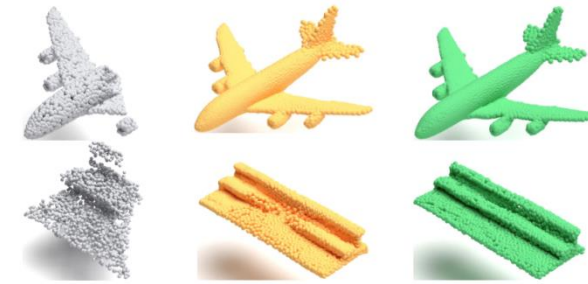


Fig.4 Visualization of ShapeNet-55 benchmark. Gray is the input. Yellow is HyperCD. Green is Landau CD.

Results

- **Training Analysis:**

- In Fig. 5, our weightedCD losses exhibit a more rapid convergence compared to HyperCD.

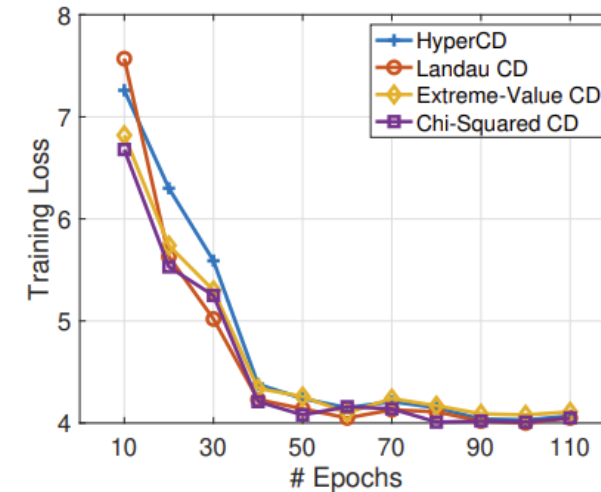


Fig. 5 Convergence comparison

- In Table.2, the training efficiency still consist with original target CD.

Loss Functions	Computation Time
HyperCD	0.4239±0.0019
InfoCD	0.4298±0.0014
LandauCD	0.4341±0.0026

Table. 2 Training efficiency comparison

- A novel loss distillation method for point cloud completion by mimicking the learning behavior of HyperCD based on weighted CD.
- An efficient and effective gradient matching algorithm to search for potential weighting functions for weighted CD.
- A bilevel optimization problem to train backbone networks, based on our iterative differentiation algorithm.
- We conduct comprehensive experiments with different datasets to demonstrate the effectiveness of weighted CD losses

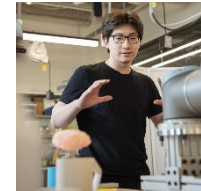
Our Team

VISLab@WPI

Fangzhou Lin: PhD student at WPI
and Tohoku University



Haotian Liu: Undergraduate student
at WPI and research intern at NEU



Haoying Zhou: PhD student at WPI
and visiting student at JHU



Songlin Hou: Research engineer at
Dell Technology



Dr. Ziming Zhang: Assistant
Professor at WPI ECE department

